

APPLICATION OF THREE-WAVE INTERACTION TO DETERMINE THE
LOCAL VELOCITIES OF MOVING MEDIA

I. A. Kolmakov, V. V. Alpatov,
N. N. Akmaev, and V. I. Volod'ko

UDC 534.222:532.574

The possibility of using a three-wave interaction to determine the local characteristics of moving media (fast-flowing processes) is examined.

Information about the local characteristics, particularly the local velocities of moving liquid and gaseous media, is of known interest to the solution of hydrogasdynamic problems as well as questions occurring in the study of fast-flowing processes: rates of combustion, relaxation processes, peculiarities of shock formation and evolution under any conditions, etc.

At this time, several methods of measuring the local velocities are known. Thus, the method of laser Doppler velocimeters (LDV), which permits contact-free measurement of local velocities by means of the Doppler frequency shift of the light scattered by particles introduced for this purpose (the LDV method), is used extensively in hydrodynamic investigations. As noted in [1] the LDV method permits measurement of the local velocities with high accuracy, $\sim 0.1\%$. In measuring the local velocities by using a thermoanemometer, diffusion transducer, Pitot tube, etc., the insertion of the detection unit in the flow of the working body is assumed, whereupon the local velocity is measured, possibly, as well as a close, but distorted picture of the velocity field. The LDV method is inertia-free, and its use does not distort the velocity field configuration in practice, in which connection it is used most extensively in high-speed aerodynamic investigations. The LDV method is elucidated in greater detail in [1] and in the literature mentioned in [1]. At the same time, when using the LDV method in optically transparent media, it is necessary to introduce light-scattering particles in a definite concentration into the medium, which excludes the possibility of its application in certain cases. In such situations the local velocities can be measured on the basis of a three-wave (three-phonon) interaction of the radiation of the ultrasonic frequency band. Here, in contrast to the LDV method, there is no need to introduce scattering particles in the moving medium, nor to use an optically transparent medium. Moreover, the local velocities can be measured through a metal wall with an appropriate construction of the radiating and detecting systems which is especially convenient in experiments with fast-flowing processes accompanied by high pressures and temperatures.

Let us examine the three-wave interaction method whose crux is the following. Two spatially diverse ultrasonic beams with nearby frequencies are directed in such a manner that their intersection would coincide with the point under investigation (the local interaction domain) of the moving medium. Because of the nonlinear wave interaction, the point of intersection is the source of secondary (quadratic) radiation [2,3], which is observed in the case of synchronized scattering upon compliance with definite conditions [formulas (2) and (3)] in the same planes where the radiators are located, i.e., in a "backward" direction. In contrast to synchronized scattering, diffraction scattering is possible only in the "forward" direction [2]. As is known [3,4], synchronized sound scattering by sound is possible in media with dispersion. At the same time, synchronized scattering is even accomplished in media without dispersion but under conditions of a flow of the medium superposed on the beam intersection domain.

Let us present the solution we obtained for the problem of sound scattering by sound in a moving medium in the form of the velocity potential (the viscosity, heat conduction, and dispersion properties of the medium were not taken into account, and plane wave interaction was assumed)

$$\varphi = \frac{A\sqrt{\Gamma_1^2 + \Gamma_2^2}}{a_1 a_2} \sin \left[\Omega t \left(1 - \frac{U}{c_0} \cos \psi \right) + \frac{\Omega r_0}{c_0} + a_3 + \arctg \frac{\Gamma_2}{\Gamma_1} \right], \quad (1)$$

Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 38, No. 6, pp. 1025-1030, June, 1980. Original article submitted March 27, 1979.

where

$$A = v_{01}v_{02}\Omega/4\pi c_0^2 r_0; \quad \Omega = \omega_1 - \omega_2;$$

$$\Gamma_1 = \frac{a_2 b (1 - \cos \alpha_2 a) - a_1 a (1 - \cos \alpha_2 b)}{a_1 a - a_2 b};$$

$$\Gamma_2 = \frac{a_2 b \sin \alpha_1 a - a_1 a \sin \alpha_2 b}{a_1 a + a_2 b}; \quad a_1 = \frac{\Omega}{c_0} \cos \psi - k_1 \cos \alpha_1 + k_2 \cos \alpha_2;$$

$$a_2 = \frac{\Omega}{c_0} \sin \psi + k_1 \sin \alpha_1 - k_2 \sin \alpha_2;$$

$\alpha_3 = \text{const}$ is a quantity defining the radiator coordinates; r_0 , spacing between the center of the beam intersection domain and the point of observation of the secondary radiation; k_1, k_2 , wave numbers corresponding to the sound waves radiated by the first and second radiators; a, b, l , linear dimensions of the intersection domain; α_1, α_2 , angles at which the radiation occurs at the frequencies ω_1 and ω_2 in a rectangular coordinate system; ψ , angle of observation of secondary radiation; v_{01}, v_{02} , amplitudes of the vibrational velocities of the first and second radiators; and c_0 , sound propagation velocity in the given medium.

Components which introduce an insignificant contribution and characterize the change in amplitude due to the removal of the secondary signal by the flow of the medium were not taken into account in (1), and moreover, distortion of the shape of the interaction domain because of superposition of the flow was not taken into account in the integration over the domain, i.e., it was assumed that $U \ll c_0$. Let us note that the passage from the velocity potential in (1) to the acoustic pressure, velocity, and density is executed in the usual manner, for instance: $P = -\rho_0 \partial \Phi / \partial t$; $\vec{v} = \text{grad } \Phi$, etc. It follows from (1) that the velocity at the point under investigation can be determined by means of the Doppler frequency shift of the secondary radiation relative to the frequency in the medium at rest, and depends on the angle of observation ψ . In solving the problem it was assumed that the flow velocity in any part of the local domain is identical and equal to U . The synchronization condition can be written as follows ($\alpha_1 = 0, \alpha_2 = \theta$):

$$\Omega = \left[\left(1 - \frac{U}{c_0} \right)^2 (\omega_1 - \omega_2 \cos \theta)^2 + (\omega_2 \sin \theta)^2 \right]^{1/2}. \quad (2)$$

This condition is satisfied for angles θ between the directions of interacting wave propagation which satisfy the equality

$$\theta^2 = \frac{2U\Omega}{\omega_2(\omega_1 c_0 - 2U\Omega)}. \quad (3)$$

Then the direction of the secondary radiation is determined by the expression

$$\text{tg } \psi = - \sqrt{\frac{2U\omega_2}{(1 - U/c_0)^2 (\omega_1 c_0 - 2U\Omega)}}. \quad (4)$$

It follows from (3) and (4) that radiation of the frequency Ω is possible for $U = 0$ only in parallel beams, i.e., superposition of the flow on the domain of interaction is equivalent to the appearance of dispersion properties in the medium.

Therefore, the effect of sound scattering by a quadratic nonlinearity permits obtaining information about the local velocities. Under real conditions the idealized situation analogous to the conditions of the problem, whose solution is presented below, is observed only in exceptional cases. However, taking account of such important factors as the dispersion, the presence of the shear viscosity, and the heat conduction of the medium is not difficult, and does not affect the crux of the question under consideration. (It can also be noted that damping of the difference-frequency wave in media with Stokes viscosity and heat conduction will occur very much more slowly than the damping of the high-frequency oscillations of the interacting beams.)

Meanwhile, it is expedient to examine such questions as the sensitivity of the three-wave interaction method, the influence of noise on the secondary signal, etc. To do this we present certain approximate data about the influence of noise on the secondary signal.

It is shown in [5] that the diminution of signal intensity in the approximation of a nondispersive medium is independent of the noise spectrum and is determined by the noise energy density. The intensity of the secondary signal varies because of nonlinear self-action

accompanying the pumping of the fundamental wave energy into the harmonic and because of interaction with the noise. Interaction between the secondary radiation and the noise will occur most effectively for noisy waves being propagated at a certain angle ($\gamma \leq \theta$) less than the angle of synchronized interaction ($\omega \gg \Omega$). The main contribution to sound absorption is introduced by thermal phonons $\gamma = 2\pi^2 c_0 / \Omega x$, whereupon $\alpha = \pi \epsilon E \Omega / 2c_0^3 \rho_0$, then the absorption coefficient is $\alpha = \epsilon^2 E \omega \tau / 2c_0^3 \rho_0$ for $\omega \tau > 1$ and $\alpha = \epsilon^2 E \omega \tau / 2c_0^3 \rho_0$ for $\omega \tau \ll 1$ (here $\epsilon = (c_p/c_v + 1)/2$, E is the noise energy, and τ is the phonon relaxation time). By using these formulas, the change in secondary signal amplitude which occurs because of secondary signal interaction with the noise can be estimated.

Let us illustrate the results of [5] presented above in an example of interaction between a signal of frequency Ω and the noise of a subsonic jet issuing from a combustion chamber. As is shown in [6], the noise level can be 20-90 dB ($2 \cdot 10^{-4}$ - $0.6 \cdot 10$ N/m²) and depends mainly on the relationship between the air and propellant mass flow rates, the combustion chamber geometry, the total discharge through the chamber, and the spectral density of the noise intensity in the chamber is determined primarily by the low-frequency components (within 10^3 Hz limits). It follows from the computations for this case that the absorption due to interaction between the secondary signal and the noise is two orders of magnitude less than the classical value (because of viscosity and heat conduction). It is assumed in this same paper [6] that the noise intensity upward over the jet can increase ten and more times. But even in this case, the noise absorption is at least an order of magnitude less than the classical value at a pressure in the chamber equal to several atmospheres.

The signal-to-noise ratio in the example under consideration is $I_\Omega/I_\omega = 5-50$ indirect proximity to the beam intersection domain for large values of $\alpha = 0.1$ cm⁻¹ of the intensities of radiators at the frequencies ω_1 and ω_2 equal to 10^2 W/cm², and spacing of $l = 50$ cm between the emitters and the intersection domain. For lesser values of α this ratio grows substantially.

The absolute sensitivity of the method, i.e., the minimal flow velocity in the local region that can be measured by this method under the condition that the difference-frequency signal being amplified is on the order of units of kHz for a passband on the order of single Hz is 10^{-2} m/sec. The relative sensitivity is 10^{-2} . The frequency meter ChZ-30, e.g., can be used as an instrument to measure the secondary radiation frequency in many cases. The scattered sound level in both kinds of scattering depends substantially on the angle of sound beam intersection. In synchronized scattering the maximum energy scattering level will be observed upon compliance with condition (3). Thus, for example, $\theta \sim 1^\circ$ for $\omega_1 = 100$ kHz, $\omega_2 = 80$ kHz, $\Omega = 20$ kHz, and $U = 1$ m/sec, i.e., for such relationships between the parameters the synchronized radiation of the frequency is possible in parallel beams in practice and in this case the very concept of locality is meaningless. If the medium possesses dispersion properties, then the synchronized scattering can occur for large values of the angles (under conditions of conservation of the other parameters).

In the case of diffraction scattering, the secondary radiation is observed in the "geometric shade" domain in the "forward" direction. Here the scattered sound level will depend on the number of moiré zones being stacked up in the sound beam intersection domain. For $\theta < \lambda/2d$ (λ is the wavelength at the frequency Ω , and d is the beam diameter), there is one zone in the beam intersection domain, the secondary wave phases differ by less than π , and maximum radiation intensity at frequency Ω is observed. As the angle θ increases, the number of zones in the beam intersection domain grows and the scattered sound level, being diminished, can fluctuate [2]. A further increase in the angle results in oscillation-free scattering, but the scattered sound level here is considerably lower and the maximum intensity of radiation at the frequency Ω will be in directions close to $\theta = 0^\circ$. For diffraction scattering, the level and direction of radiation at the frequency Ω are slightly responsive to the change in shape of the interaction domain, in contrast to synchronized scattering. Diffraction scattering is possible even for large values of the angles θ . Thus, sound is scattered by sound in (5) for $\theta = 90^\circ$ and $\omega_1 \gg \omega_2$, the difference frequency can hence be in a direction practically coincident with the direction of frequency radiation.

The accuracy of measuring the local velocities by this method will be determined mainly by the accuracy in measuring the Doppler frequency shift Ω and the accuracy in measuring the spatial arrangement of the radiator-detector system (in addition to the error of the measuring apparatus). In this connection, high accuracy in measuring velocities by this method ($\sim 1\%$) can be expected, especially in narrow ranges of measuring the parameters of the medium.

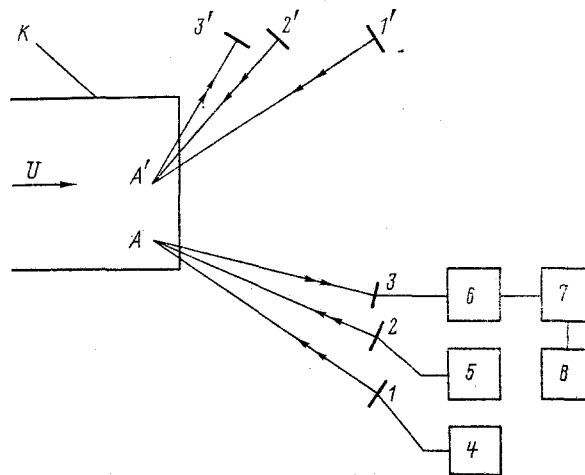


Fig. 1. Block diagram illustrating the use of three-wave interaction to determine local velocities: K) combustion chamber; A, A') domains in which the local flow velocities of the combustion products are determined; 1, 2, 1', 2') radiators of the frequencies ω_1 and ω_2 ; 3, 3') detectors; 4, 5) signal generators at the frequencies ω_1 , ω_2 ; 6) electrical filter to extract the difference frequency $\Omega = \omega_1 - \omega_2$; 7) frequency signal amplifier; 8) oscilloscope.

A diagram of one possible version for applying the proposed method to determine the local velocities of the combustion products in the experiment presented above, say, is presented in Fig. 1. The local velocities are here determined at the points A and A'; 1, 2 and 1', 2' are radiators at the frequencies ω_1 and ω_2 ; 3, 3' are detectors (e.g., from barium titanate [2]). Radiation of frequency Ω is incident from the detector onto the electrical filter, the signal is later amplified and delivered to the oscilloscope, where it is analyzed, or to a frequency meter. In certain cases an interference method may be used for a more accurate measurement of the Doppler shift. It is assumed that the nonlinearity of the apparatus in such experiments is several orders of magnitude less than the nonlinearity of the medium under investigation.

In conclusion, let us present other examples of using the method proposed. The interaction between shock and sound waves is studied in [7]. Experiments showed that the passage of a sound wave through a shock results in magnification and change in the sound wave frequency depending on the Mach number of the shock. It is noted that the magnification can be so great that the acoustic waves become finite-amplitude waves and a weak shock is formed because of their mutual interaction. In connection with these results, the possibility, in principle, of using the method to investigate the local characteristics of a shock front and the zone behind the front, in particular to investigate the part of the front near the wall along which shock propagation occurs, as well as relaxation processes accompanying the propagation of such waves, is obvious.

In principle, the local temperature fluctuations originating during vibrational combustion can be investigated by this same method. Thus, if the vibrational combustion occurs under such conditions at which it can be assumed that the sound propagation velocity has primarily a temperature dependence, then the temperature fluctuations in a local domain will be accompanied by corresponding changes in the sound propagation velocity in this domain and in the lengths of the interacting waves of the frequencies ω_1 , ω_2 . Consequently, even the direction of the secondary radiation will also vary, "taking off" after the temperature changes. The angle of deviation of the secondary radiation direction is proportional to the temperature change. Temperature fluctuations in such cases can be determined by an interference method also since the temperature change in a local domain specifies a change in the refractive index, i.e., results in a shift in the interference fringes by which the temperature change can indeed be found.

In certain cases, the pressure change in a local domain can also be determined by the change in secondary radiation intensity.

LITERATURE CITED

1. Methods of Laser Doppler Diagnostics in Hydroaerodynamics [in Russian], *Énergiya*, Moscow (1978).
2. V. A. Zverev and A. I. Kalachev, "Sound radiation from the domain of intersection of two sound beams," *Akust. Zh.*, 15, No. 2 (1969).
3. O. V. Rudenko and M. I. Soluyan, *Theoretical Principles of Nonlinear Acoustics* [in Russian], Nauka, Moscow (1975).
4. L. K. Zarembo and V. A. Krasil'nikov, *Introduction to Nonlinear Acoustics* [in Russian], Nauka, Moscow (1966).
5. O. V. Rudenko and A. S. Chirkin, "Theory of nonlinear interaction between monochromatic and noise waves in weakly dispersing media," *Zh. Eksp. Teor. Fiz.*, 67, No. 5(11) (1974).
6. A. N. Abdel'khamid, D. T. Harrier, T. G. Pullet, and M. Summerfield, "Noise characteristics of a subsonic jet issuing from a combustion chamber," *Raketrn. Tekhn. Kosmonav.*, 12, No. 3 (1974).
7. M. A. Ibragim, A. I. Klimov, and F. V. Shugaev, "Sound wave interaction with a shock," *Akust. Zh.*, 24, No. 4 (1978).

DRAG REDUCTION BY CATION SURFACTANTS: THE RELATION TO
PHYSICOCHEMICAL AND MICELLAR CHARACTERISTICS

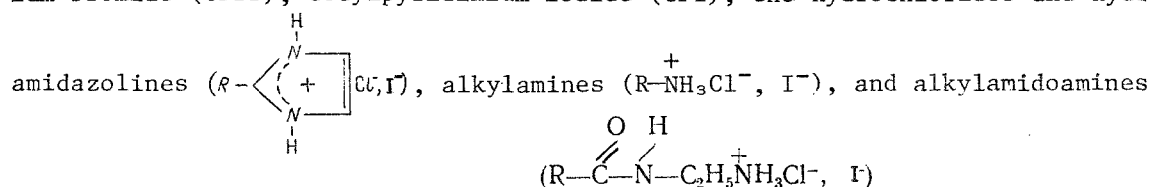
I. L. Povkh, A. I. Serdyuk,
A. V. Naumov, and N. P. Kovalenko

UDC 532.517.4:661.185.23

It is shown that systems containing the cationic surfactant α -naphthol begin to reduce the hydrodynamic resistance at concentrations of 0.02-0.05%, at which solutions contain nonspherical micelles of a particular size.

We have previously shown [1] that the combination of cetylpyridinium chloride (CPCl) with α -naphthol with a 1:1 ratio of the components reduces the drag in the turbulent flow of a liquid (water). Here we consider the drag reduction for water caused by various cationic surfactants in the presence of α -naphthol, and we derive a relationship between the reduction in the drag and the physicochemical parameters of the solutions.

We chose the cationic surfactants cetyl- γ -picolinium bromide (C- γ -PicBr), cetylpyridinium bromide (CPBr), cetylpyridinium iodide (CPI), the hydrochlorides and hydroiodides of alkyl-



in which the lengths of the alkyl chains R were 12 and 16 carbon atoms.

The drag was measured with a glass tube of length 4 m and diameter 5.45 mm for a Reynolds number of 10,000. The drag reduction ($\Delta\lambda/\lambda$) was calculated from

$$\frac{\Delta\lambda}{\lambda} = 1 - \frac{\tau_2^2}{\tau_1^2} \frac{\Delta P_2}{\Delta P_1}$$

The viscosities of the solutions were determined with a BPZh-2 capillary viscometer, while the density was determined by a pycnometer method. The IR spectra of these surfactants dissolved in carbon tetrachloride with and without α -naphthol were recorded with a UR-20 spectrophotometer in the OH stretching region. The optical density of Sudan II (a water insoluble dye solubilized by the surfactants) was measured with an SF-16 spectrophotometer at 550 nm.

The measurements (Fig. 1) show that systems such as C- γ -PicBr- α -naphthol, CPBr- α -naphthol, and CPI- α -naphthol reduce the drag at the very low overall concentration of